Mr. López. Please, let me know of errors and suggestions: *mathdoc4u*@ *yahoo.com*

As in everyday life, you can **add and subtract like things**: pears with pears, pencils with pencils, kwanzas with kwanzas, sevenths with sevenths, **z**'s with **z**'s, $((a^3)(b^2))'s$ with $((a^3)(b^2))'s$ ...

Fractions (Latin: fractus, "broken") are numbers expressed as the ratio of two numbers, representing how many parts of a whole (That slice is (1/8) of the water melon, 9 quarters pay a train ride). The number "on the top" is called **numerator** (Latin: "says how may") and the one on the bottom is the **denominator** (Latin: "says what it is" (the thing's name)). **Equivalent fractions** are those that represent the same value: (1/2) = (2/4) = (3/6); (6/5) = (12/10). If you divide (6/5) using a calculator, the value you get will be exactly the same as when you divide (12/10); and (1/2) the same as (50/100) (43/86) ... In fact the **decimal number** you get from your calculator is just another way to write the exact same number. An **improper fraction** is one in which the numerator is greater than the denominator, examples: (7/3), (12/10); they can be rewritten as **mixed numbers**: (7/3) = 2 (1/3) = 1 (4/3) and (12/10) = 1(2/10) = 1(1/5). A mixed number represents a division in which the integer result is the quotient, the remainder is the numerator and the denominator is the divisor.

There are three **<u>cases of addition and subtraction of fractions</u>**:

I._ when the **denominators are the same** (like fractions): the denominator of the result is the same and you just add or subtract the numerators: (1/3) − (5/3) = −(4/3), (6a/ xy^2) + (3b/ xy^2) = (6a + 3b) / xy^2 ;

II._ when the **denominators are not equal but one of them is a multiple of the other**, you use the greater denominator (the multiple) as a **common denominator** and rewrite the other fraction so that you can add them as in case I: (1/10) + (3/5) = (1/10) + (6/10) = (7/10) (because (6/10) and (3/5) are equivalent, (3/5) = ((3x2)/(5x2)) = (6/10)); (2/7) − (6/28) = (8/28) − (6/28) = (2/28) = (1/14) ((2/7) = (8/28), (2/7) = ((2x4)/(7x4)) = (8/28));

III._ when the **denominators are neither equal nor is one of them mutiple of the other**, you have to use the **least common multiple** (or "lowest common denominator", with which you effectively rewrite both fractions as like ones): (2/3) + (1/4) = ((8/12) + (3/12)) = ((8+3)/12) = (11/12); (5/y) − (10/z) = (5z/z) − (10y/z) = (5(z−2y)/z).

In order to **<u>multiply fractions</u>**, just multiply both numerators to get the numerator of the product and both denominators to get denominator: (7/9) x (4/5) = ((7x4)/(9x5)) = (28/45). Multiplying fractions, say ((1/2) x (1/3)) is the same as saying "half of one third" (you have one third of something and take half of it). **<u>Dividing fractions</u>** is a two-step process: 1) turn the division into a multiplication; 2) flip the second fraction; then you complete the resulting multiplication of fractions: (4/3) % (7/6) = (4/3) x (6/7) = 8/7 Notice that in order to **<u>multiply or divide mixed numbers</u>** you must first express them as improper fractions, which you do by multiplying the denominator by the quotient and the result is added the numerator: 2 (1/3) = ((2 x 3) + 1)/3 = (7/3), 1 (1/5) = (((5 x 1) + 1)/5) = (6/5) = (12/10)

<u>Percentages</u> are equivalent to fractions which denominator is 100 ("per cent"). 25% is the same as saying (25/100) = (1/4) (a "quarter" (a fourth) is 25% of a dollar). 75% of 200 means you have 200 units of something and want to know how much would that amount at a rate of (75/100): 200 x (75/100) = (200/1) x (75/100) = 2 x 75 = 150. **W** x **%** = **P**: the principal **W**hole times the **P**ercent is the **P**ortion (which may be greater if the percentage is greater than 100%)

<u>Algebra</u>: To **add or subtract like terms** they should first be grouped on the same side of the equation, taking them from one side to the other of an equation or inequality using the **inverse operation**. Addition and subtraction are inverse operations of each other, so are multiplication and division, and exponentiation and root extraction of numbers: (q/7) = 2 => (implies) q = 2x7 = 14 (7 divides q on the left, therefore it is taken to the right side of the equation by multiplying it to 2). 3**a** and 5**a** are like terms in the equation: (3**a** + 7) = 5**a**; "3**a**" is positive on the left side, osea passed as a negative term to the right.

Always **write algebraic expressions canonically**, that means, starting with the numerical factor and then the algebraic terms alphabetically: $((c^2)(b^4)(c^7)3(b^{-1})(5))$ = $(15(b^3)(c^9))$

When you **multiply, divide or operate on** differing **signed terms across parentheses**, the result is negative (otherwise it is positive).

Any numeric or algebraic expression multiplied or divided by 1, or raised to the power of 1 is the same term: 5x1=5, (18/1) = 18, $(3/5)^1$ = (3/5), $(a+4)^1$ = (a+4). The result of multiplying any term by 0 is 0: (4x0)=0, x^2 · 0 = 0 (the division by 0 is undefined)

A decimal number is a fraction which denominator is a power of 10: (0.375 = 375/1000 = $(375/10^3)$ = ((3x125)/(8x125)) = (3/8)). T**o divide a decimal number** by a **power of 10** move the decimal point to the left as many times (introducing zeros if necessary) **as the power of 10**. While **multiplying decimal numbers** you **move the decimal point to the right**: (3/8) x 10^2 = 0.375 x 10^2 = 0.375 x 100 = 37.5

<u>Exponentiation and root extraction</u>: A base number elevated to an exponent tells you how many times the base number is used as a factor: 5^2 = 5 · 5 = 25, $(-2)^3$ = (−2) · (−2) · (−2) = −8, 7^4 = 7·7·7·7 = 2,401, $(y^2z)^3$ = (y^2z) · (y^2z) · (y^2z) = y^6z^3 , a^n = a·a·a·...·a (base, **a,** is factored **n** (the exponent) times) Any number raised to the power of zero (including zero itself) equals to one ($a^0=1$): 2^0 =1, $(57)^0$ =1, $(6-6)^0$ =1, $(a+b)^0$ = 1. In order **to multiply algebraic or numerical expressions with the same base** you add the exponents: (3^2x3^4) = $(3^{(2+4)})$ = 3^6 = 729; only if they have the **same exponent** you **multiply the bases**: (7^23^5) = $(7^23^23^3)$ = (21^23^3) = 441 x 27 = 11,907

Priority Rules for Arithmetic: Always work progressively considering each operation step by step:

1) from the innermost parentheses outward
2) exponents and roots
3) multiplication and division
4) addition and subtraction

$(3\cdot(a+4)+2a)^2$ = $(3a+12+2a)^2$ = $(5a+12)^2$ =(5a + 12) **x** (5a + 12)

Steps: the innermost parentheses is enclosing (a + 4), we know we can not add a to 4, so we must consider the immediate operation, which is multiplying each of the terms inside of that parentheses by 3, then we can add the 3a with 2a ...

Any **number or algebraic expression raised to a negative exponent** equals to 1 over the number to the same positive exponent: a^{-n} = 1/ a^n , 3^{-2} = 1/ 3^2 = (1/9), $(x^2-y^2)^{-1}$ = 1/ (x^2-y^2)

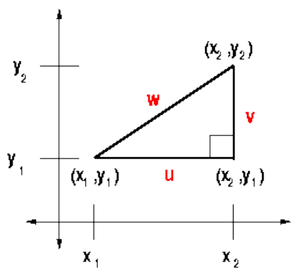
$(a/b)^n$ = (a/b) · (a/b) · ... · (a/b) = (a · a · ... · a) / (b · b · ... · b) = (a^n/b^n) (multiplying the fraction)

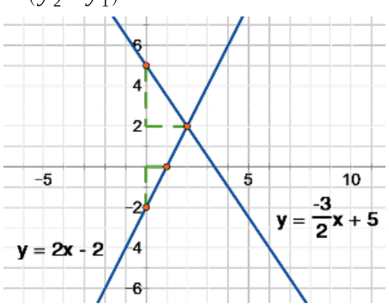
Finding the root of a number is the inverse operation of raising it to a power: since 5^2 = 25, the square root of 25 es 5 ($\sqrt[2]{25}$ = 5), la cubic root of 125 is 5 ($\sqrt[3]{125}$) because 5^3 = 125 and the forth root of z^8 ($\sqrt[4]{z^8}$) is z^2 because $z^2\cdot z^2\cdot z^2\cdot z^2$ = z^8

Measures of <u>length</u>: 1 foot = 12" (inches), 1 yard = 3 feet = 36", 1 mile = 1.760 yards; **<u>time</u>**: 1 day = 24 hours, 1 hour = 60 minutes, 1 minute = 60 seconds; **<u>weight</u>**: 1 pound = 16 ounces. When adding and subtracting positional measurements whose conversion is not expressed in base 10, you must bear in mind that if you take a unit of the greater immediate position this could amount to add 12, 16, 60; 7 or 24.

Given a set of **measurable events** (rainfall, calorie and fat intake, a batter reaches first base or not), you can find the **average** or **arithmetic mean** dividing the sum of each of the values observed by the total number of events. The **median** is the value of so much data before and after, and **fashion** is the most frequently occurring value.

The sum of the internal angles of any triangle is 180° (180 degrees). The area of any geometric figure can be totaled by successive **triangulations**. When two lines intersect they form two pairs of **opposite** (to one another) and **adjacent** (sharing a common side) angles in reference to to the apex (the lines' cut point). The sum of the measures of adjacent angles is 180°. The sum of the sides of a closed plane figure is its **perimeter**. In right triangles, **the sum of the squares of the two legs equals the square of the hypotenuse** (**Pythagorean theorem**), which is used to find the distance between two points on a coordinate system: $w^2=u^2+v^2$ = $(x_2-x_1)^2$ + $(y_2-y_1)^2$





There is a **<u>correspondence</u>** between **<u>algebraic funtions</u>** and (their) **<u>graphs</u>** in a coordinate system. A straight line, determined by its intersect with the y-axis (**b,** when x = 0) and its slope (m) is written as a linear ecuation: y = f(x) = **m**·x + **b;** for example: y=–(3/2)·x+5. A parabola, f(x) ≈ x^2 is the curve described by the flight of batted baseballs and launched missiles. To solve a system of 2 equations with 2 variables, you should: 1) **write both equations as functions (y=f(x))**, and 2) (since y = y) you then equate both functions and solve the equation: 1) y + 6 = 3x, 2) y + x = 6. Rewriting §1; y = 3x – 6; §2 y = 6 – x, so 3x – 6 = 6 – x, ...

<u>To solve word problems</u>: 1) **read the question**, 2) **read carefully** and **mentally recreate the problem**, 3) determine the **key phrases** (**irrelevant aspects** and red herrings), 4) analyze **relationships between the data** and §1, 5) **translate** §4 to **mathematical relationships**, **identify the formulas** to use, 6) make **intermediate calculations** and **conversions**; 7) **work out** the solution, 8) **check** result

Formulas for the **p**erimeter and **a**rea of plane figures based on their **s**ide, **b**ase, **h**eight and **r**adius:

Δ triangle **P**: $(s_1+s_2+s_3)$ **A**: $(1/2)(b\cdot h)$

parallelogram/rectangle **P**: $(2\cdot(b+h))$ **A**: $(b\cdot h)$

☐ square **P**: $4\cdot s$ **A**: s^2

trapezoid **P**: $(s_1+b_1+s_2+b_2)$ **A**: $(1/2)((b_1+b_2)\cdot h)$

O circle **P**: $2\pi\cdot r$ **A**: $\pi\cdot r^2$ $(\pi\approx 3.14)$

Surface **a**rea and **v**olume of spatial figures:

cube **A**: $6\cdot s^2$ V: s^3 (**s**ide of edge)

cylinder **A**: $2\pi\cdot r\cdot(r+h)$ V: $\pi\cdot h\cdot r^2$

cone **A**: $\pi\cdot r\cdot(r+h)$ **V**: $(\pi/3)\cdot r^2\cdot h$ (slant **h**eight)

sphere **A**: $4\cdot\pi\cdot r^2$ V: $4\cdot\pi\cdot r^3/3$